

What you'll Learn About

- How use slopes to create a solution to a differential equation

Given the function $y = \frac{1}{2}x^2$. Find what $\frac{dy}{dx} = x$

At each grid point representing integers, calculate the value of the derivative and draw a short line segment with that slope.

$$x=0 \quad \frac{dy}{dx} = 0$$

$$x=1 \quad \frac{dy}{dx} = 1$$

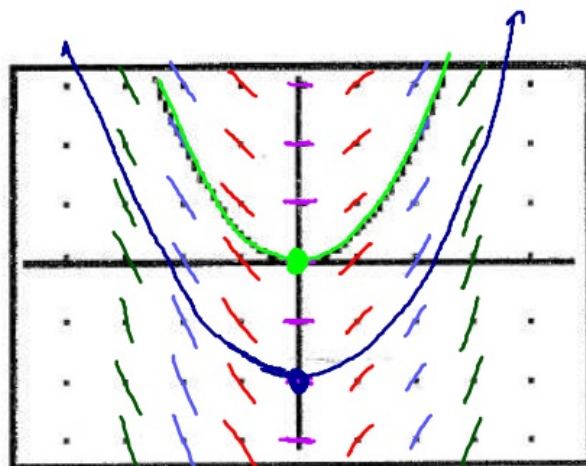


Figure 1

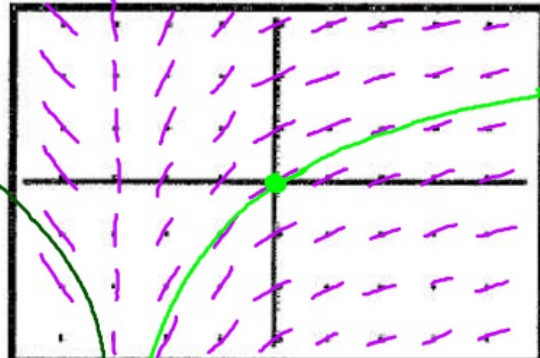
A) What family of functions seems to match all the slope fields?

Parabolas

B) What is an initial condition of the function graphed?

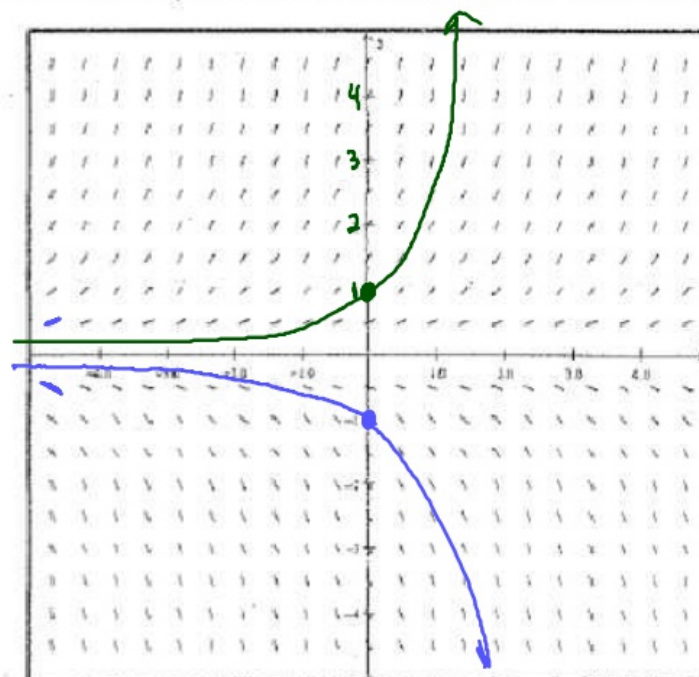
4. If $\frac{dy}{dx} = \frac{1}{x+3}$, sketch the slope field

$$x=0 \quad \frac{dy}{dx} = \frac{1}{3}$$



$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{x+3} \\ \int dy &= \int \frac{1}{x+3} dx \\ y &= \ln|x+3| + C\end{aligned}$$

Given the slope field shown below answer the following questions.



- Sketch a path of the unique solution that passes through $(0, 1)$.
- Sketch a path of the unique solution that passes through $(0, -1)$.
- What familiar functions do these resemble? *exponential*

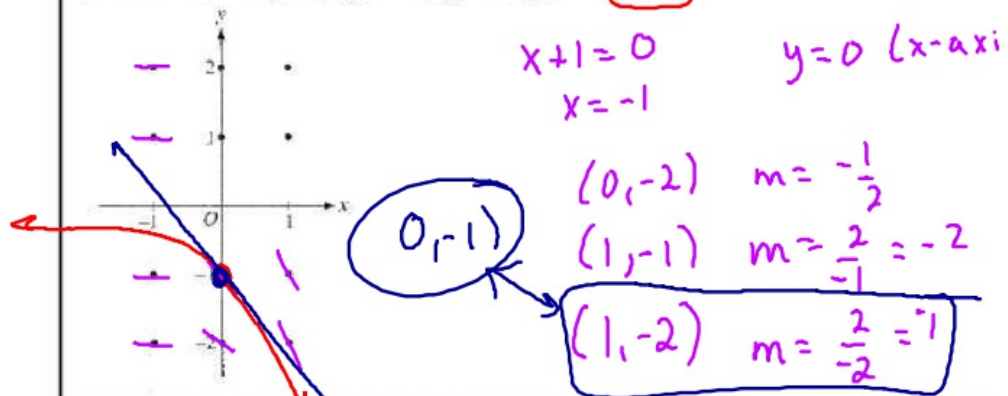
d) Given $\frac{dy}{dx} = y$, verify your guess analytically.

$$\begin{aligned} \frac{dy}{y} &= \frac{y}{y} dx \\ \int \frac{1}{y} dy &= \int 1 dx \\ \ln|y| &= x + C \\ y &= e^{x+C} \quad y = Ae^x \end{aligned}$$

$$(-1, 0) \quad m = \frac{0}{0}$$

5. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$

a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated and for $-1 < x < 1$, sketch the solution curve passing through the point $(0, -1)$



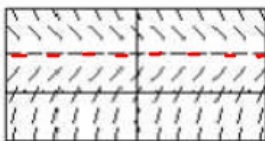
b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$

$$y = -1 - |x|$$

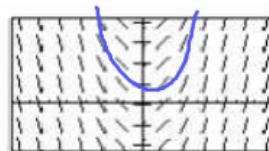
c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

Match the following differential equation with the correct slope field

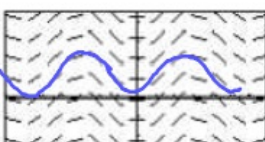
(A)



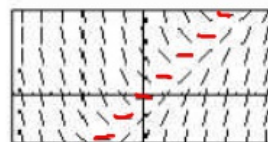
(B)



(C)



(D)



7. $\frac{dy}{dx} = \sin x$

$y = -\cos x + C$

8. $\frac{dy}{dx} = x - y$

$0 = x - y$
 $x = y$

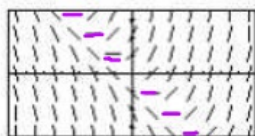
9. $\frac{dy}{dx} = 2 - y$

$y = 2$
 A

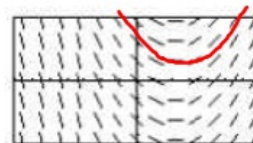
10. $\frac{dy}{dx} = x$

$y = \frac{1}{2}x^2$
 B

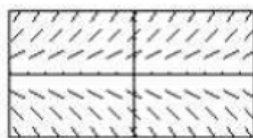
(A)



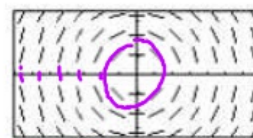
(B)



(C)



(D)



11. $\frac{dy}{dx} = 0.5x - 1$

B

12. $\frac{dy}{dx} = 0.5y$

C

13. $\frac{dy}{dx} = -\frac{x}{y}$

$y = 0$ (x-axis)
 D

14. $\frac{dy}{dx} = x + y$

A